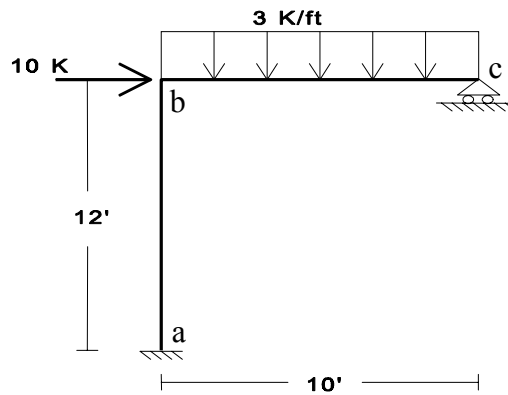


Frame Example by Slope Deflection

Objectives:

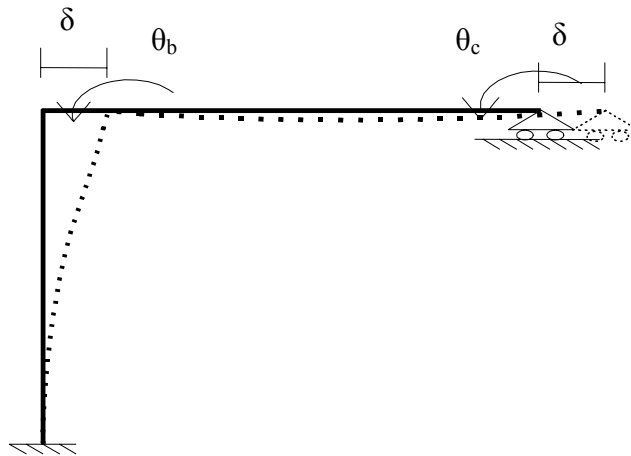
1. Apply slope deflection equations to a frame.

The following structure will be solved:



Slope Deflection – Frame Example

The displaced shape is drawn to verify that the two displacements are the required independent DOF.



DOF for Frame Example

Summing moments about joint b for θ equation. The resulting equation is:

$$M_{ba} + M_{bc} = 0$$

The equations for these moments are (units are Kips and inches):

Substituting gives:

$$0.0611EI\theta_b + 0.0166\theta_c + 0.000289EI\delta + 300 = 0$$

Summing moment about c, we get:

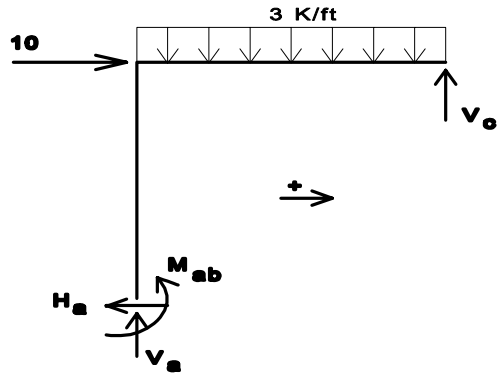
$$M_{cb} = 0$$

The required member equation is:

The final equation is:

$$0.0166EI\theta_b + 0.0333EI\theta_c - 300 = 0$$

The second equilibrium uses a free body of the entire structure and summation of horizontal forces. Displacement unknowns require a summation of forces ($\sum F_x = 0$ or $\sum F_y = 0$) or a global moment summation.

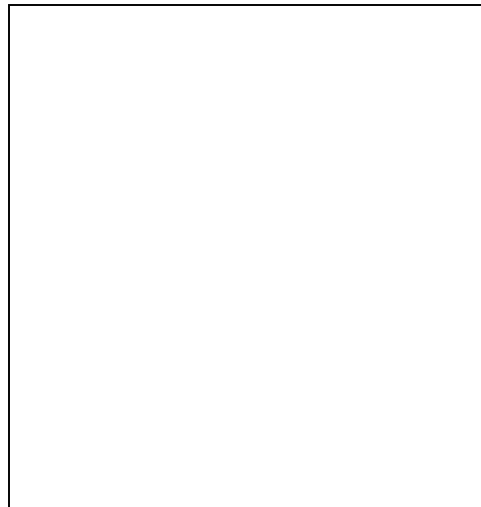


Structural Freebody - Equilibrium

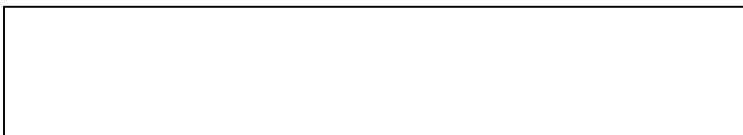
Summing forces in the x direction we get:

$$10 - H_a = 0$$

We need an equation with **M**, we have **H**. We must take another free body to get an equation with **H** and **M**. Typically, we use a single member:



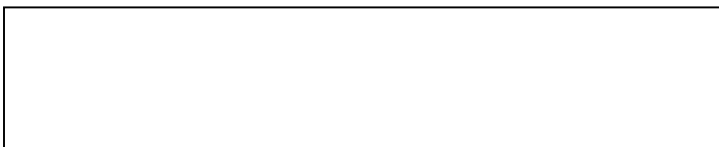
Summing moments about point b;



Solving for H_a we get:

$$H_a = \frac{M_{ba} + M_{ab}}{144}$$

Substitute into the horizontal equilibrium equation.



We need M_{ab} and M_{ba} to substitute. The equation for M_{ab} is;

$$\frac{2EI}{L} \theta_b - \frac{6EI}{L^2} \delta - \frac{4EI}{L} \theta_b - \frac{6EI}{L^2} \delta = 0$$

Reducing the expressions we get the final form of the equilibrium equation as;

$$1440 - 0.04166EI \theta_b - 0.0005787EI \delta = 0$$

If we cast the two equilibrium equations into matrix form we have:

$$\begin{bmatrix} 0.06111 & 0.01666 & 0.000289 \\ 0.01666 & 0.0333 & 0 \\ 0.04166 & 0 & 0.0005758 \end{bmatrix} \begin{bmatrix} \theta_b \\ \theta_c \\ \delta \end{bmatrix} = \begin{bmatrix} -300 \\ 300 \\ 1440 \end{bmatrix}$$

Using values of $E=29000 \text{ K/in}^2$ and $I = 400 \text{ in}^4$ and solving we get:

$$\begin{Bmatrix} \theta_b \\ \theta_c \\ \delta \end{Bmatrix} = \begin{Bmatrix} 0.003173 \\ 0.002364 \\ 0.445 \end{Bmatrix}$$

These displacements can be plugged into the slope deflection equations to solve for the moments.