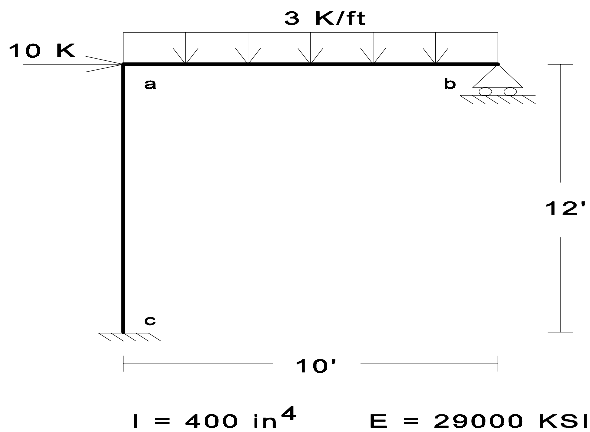
Stiffness By Definition – Frame Example

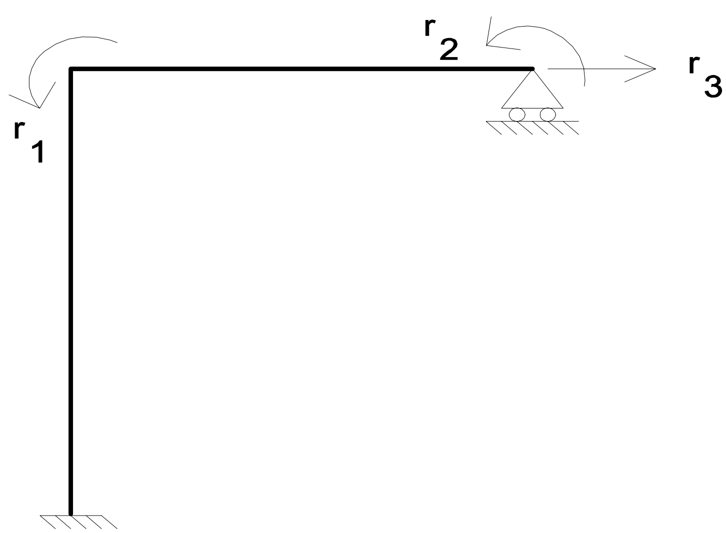
Objectives:

1. Define the required independent DOF for a structure
2. Draw the free bodies and calculate the equilibrium forces on a frame
3. Calculate the stiffness matrix for beam structures (by definition)

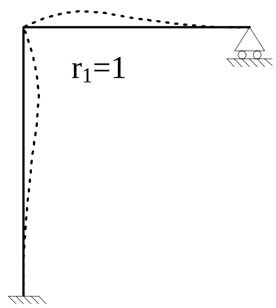
Given the following structure, find the global stiffness matrix using the stiffness by definition method.



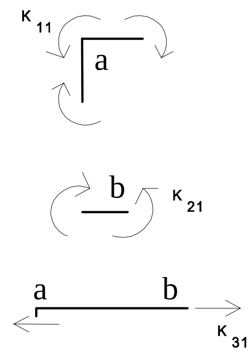
The first step is to define the DOF. All possible DOF will be used ad given below. Note that DOF 2 could be eliminated provided the modified slope deflection equations and appropriate displaced shapes are used.



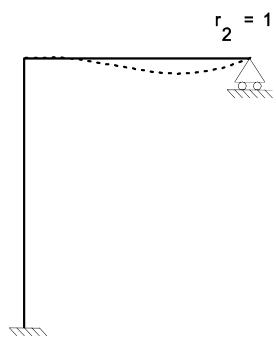
Next, a unit displacement is applied to each DOF, holding the others to zero. A free body of each joint is drawn in order to use the summation of moments as an equilibrium equation to find the rotational stiffness term. The free body of the horizontal member is used for the third equilibrium equation, summation of horizontal forces, to find the translational stiffness term. The next three figures give the displaced shapes and free bodies.



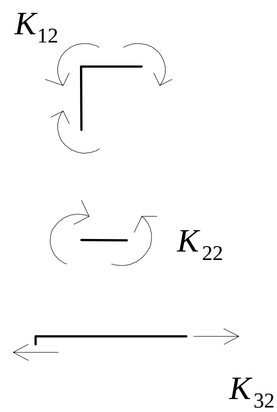
Unit Displacement DOF 1



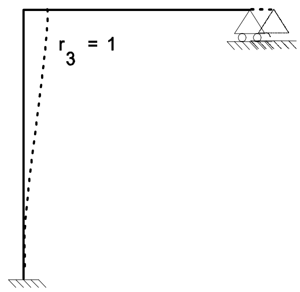
Free Bodies

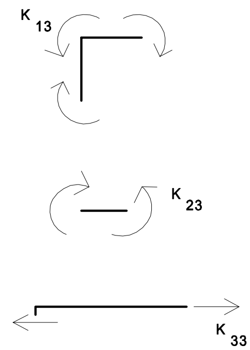


Unit Displacement DOF 2



Free Bodies





Unit Displacement DOF 3 — Free Bodies

From the free body diagrams, we can directly assemble the stiffness matrix. It is:

$$K = \begin{bmatrix} \frac{4EI}{144} + \frac{4EI}{120} & \frac{2EI}{120} & \frac{6EI}{144^2} \\ \frac{2EI}{120} & \frac{4EI}{120} & 0 \\ \frac{6EI}{144^2} & 0 & \frac{12EI}{144^3} \end{bmatrix}$$