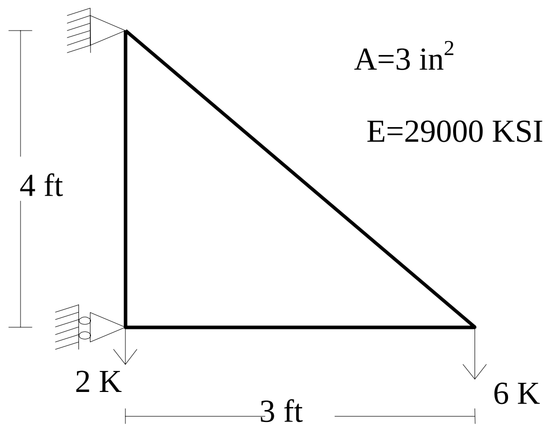
Stiffness by Definition – Truss Example

Objectives:

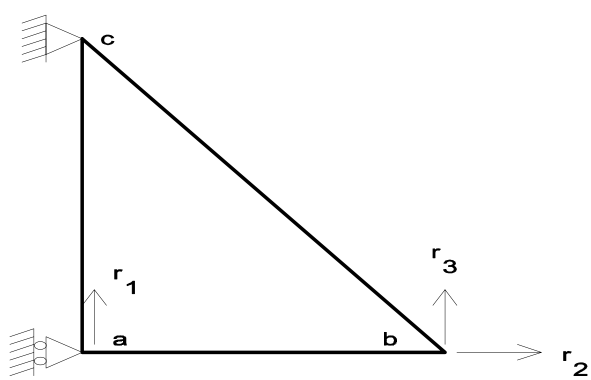
1. Define the required independent DOF for a structure
2. Draw the free bodies and calculate the equilibrium forces on a frame
3. Calculate the stiffness matrix for truss structures (by definition)

Find the stiffness for the following truss structure.



1) Define the DOF for the structure.

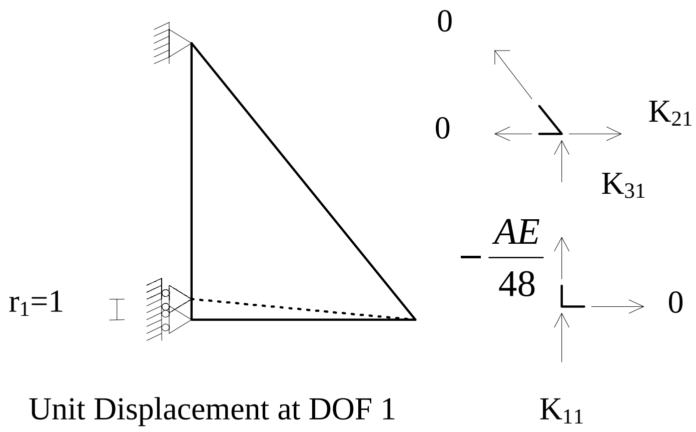
For truss structures there are only two possible DOF, translation in the plane. For the X_Y coordinate systems, this would be translations in the X and Y direction. Truss members DO NOT HAVE rotational stiffness and therefore a node MUST not have a rotational DOF. The following DOF for the structure will be used.



Global DOF Definitions

2) Apply a unit deformation at each DOF and find the forces required to hold that deformed shape.

The required displaced shape for DOF 1 is:

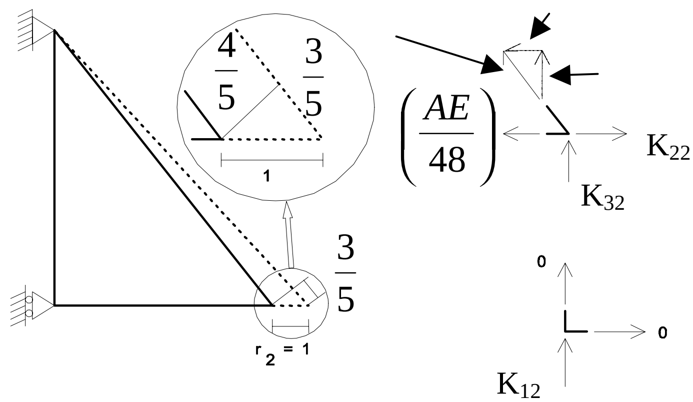


Unit Displacement at DOF 1

Joint Free Bodies

Use a free body of the joint and equilibrium to get stiffness.

Displacement perpendicular to a member causes no force. The next deformed shape is:



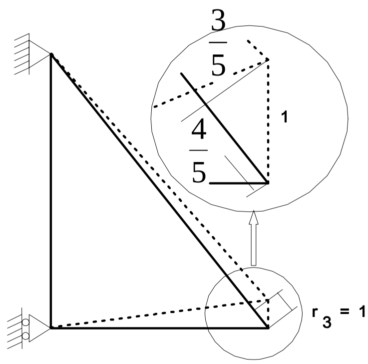
Unit displacement at DOF 2

Joint Free Bodies

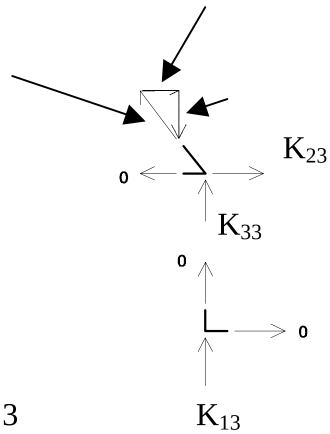
Need the amount of stretch (or shrinkage) each member goes through.

Due to small displacements, we assume that the slope of the deformed member is equal to the original member. We draw a perpendicular line from the end of the original member through the new position. The length beyond this line is the amount of extension.

The final displaced shape and free bodies are:



Unit Displacement at DOF 3



Joint Free Bodies

Gathering all the terms into a final matrix we have:

$$K = \begin{bmatrix} \frac{AE}{48} & 0 & 0 \\ 0 & \frac{AE}{36} + \frac{AE}{60}\frac{3}{5}\frac{3}{5} & \frac{AE}{60}\frac{3}{5}\frac{4}{5} \\ 0 & \frac{AE}{60}\frac{3}{5}\frac{4}{5} & \frac{AE}{60}\frac{4}{5}\frac{4}{5} \end{bmatrix}$$

Substituting in for A and E we get the final form:

$$K = \begin{bmatrix} 1812.5 & 0 & 0 \\ 0 & 2938.67 & 696 \\ 0 & 696 & 928 \end{bmatrix}$$