

Direct Stiffness using full a matrix

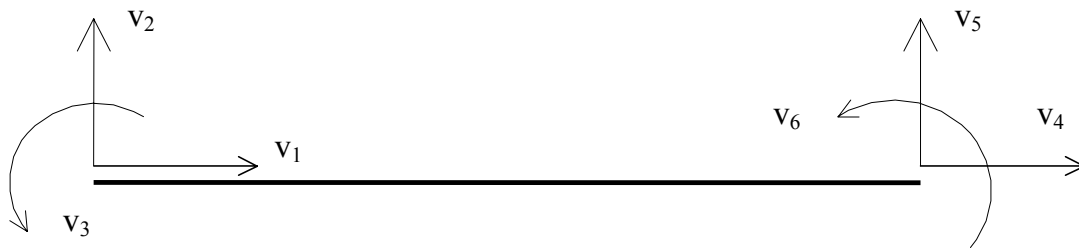
Objectives:

- 1) Define the transformation matrix
- 2) Form the transformation matrix for an element

Stiffness Transformation Matrix

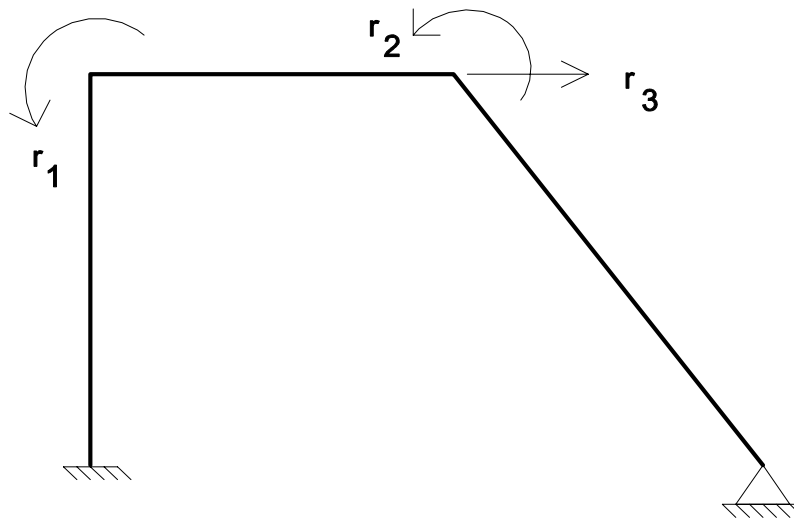
Definitions and derivations for the required equations.

\mathbf{v} = element deformations



Local Element DOF Definitions

If we look at the same structure as previous examples, we also have a set of global displacements or degrees of freedom for the structure, \mathbf{r} .



Direct Stiffness Example Structure

For any element in the structure, there exists a relationship between the element DOF and the global DOF. This relationship can be expressed as:

$$\mathbf{v} = \mathbf{a} \mathbf{r}$$

Where \mathbf{a} is defined as the global to local transformation matrix. The \mathbf{a} matrix converts a set of global (structural) displacements, \mathbf{r} , into the local or element displacements, \mathbf{v} , for a particular element.

We can define a set of element forces, \mathbf{S} , that correspond to the element deformations. There is a one to one correspondence between the element deformations and element forces. Therefore the element forces are defined as:



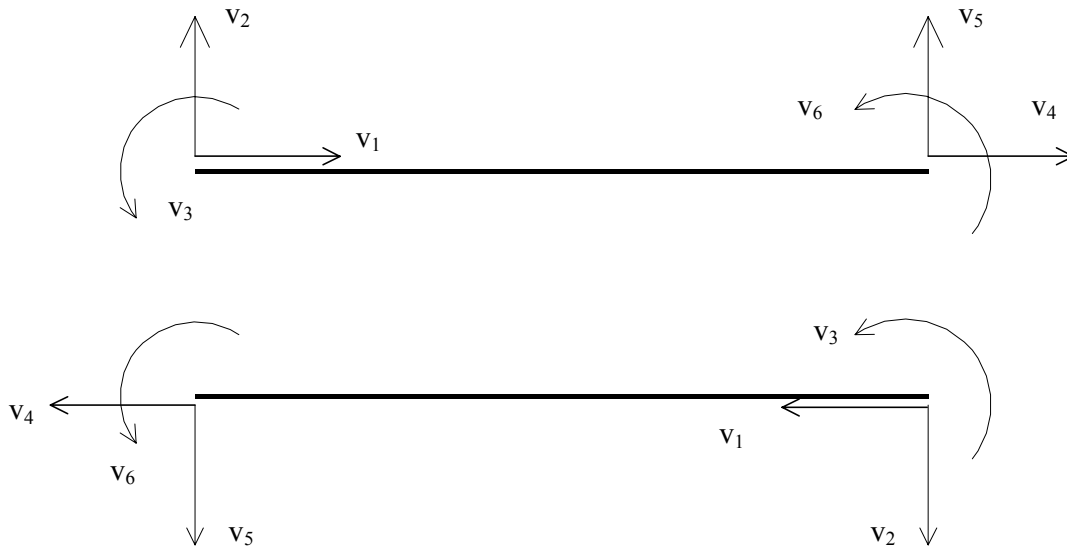
Local Element Force Definitions

A similar equation can be written that relates the element forces, \mathbf{S} , to the global forces, \mathbf{R} .

$$\mathbf{R} = \mathbf{a}^T \mathbf{S}$$

Definition of the \mathbf{a} matrix

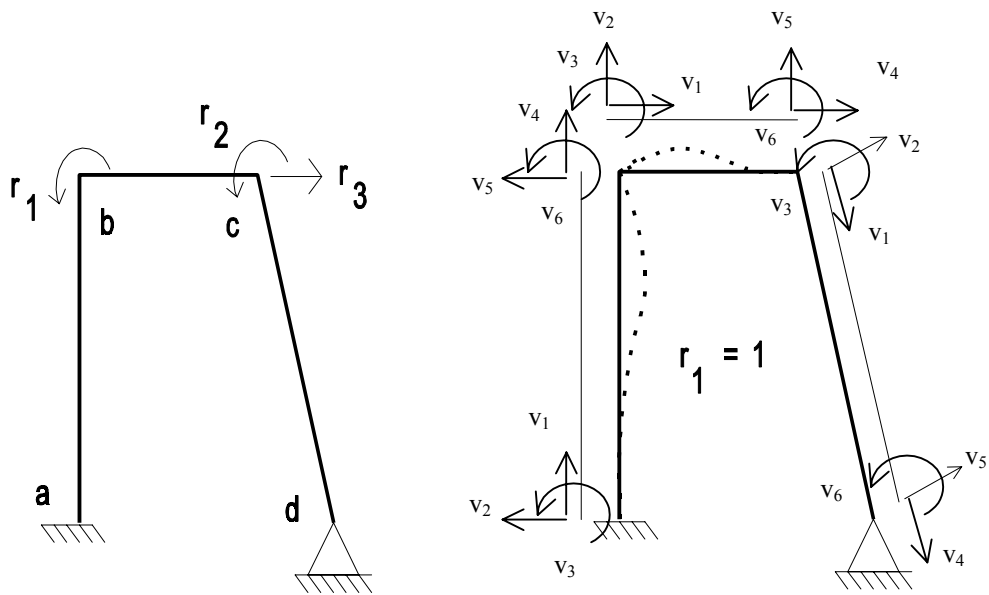
We need to define a local coordinate system. There are two possibilities:



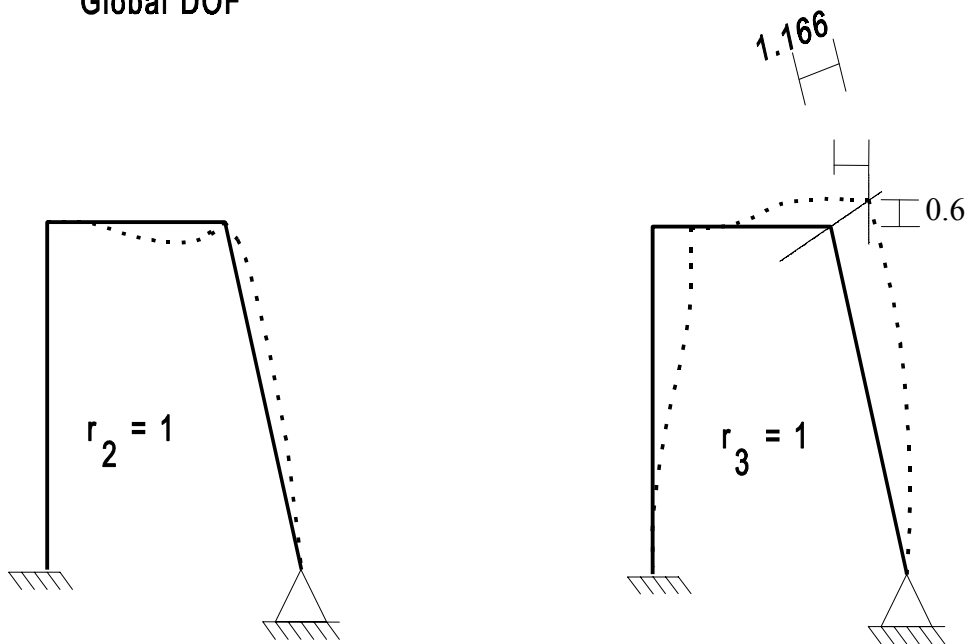
Possible Local Element Definitions
(Horizontal Element)

Either choice is equally acceptable. The only difference is that the results will have different signs. There is one local (element) coordinate system and an \mathbf{a} matrix for each element. The size of \mathbf{a} is the number of element DOF (EDOF) by the number of global DOF (GDOF). For our problems each \mathbf{a} matrix is 6 by 3.

Using the same process as deriving the stiffness matrix by definition, we apply a unit displacement to one DOF leaving all others ZERO. Each displacement gives a column of the \mathbf{a} matrix, just like the stiffness matrix.



Global DOF



Local Element Definitions and Unit Displacements

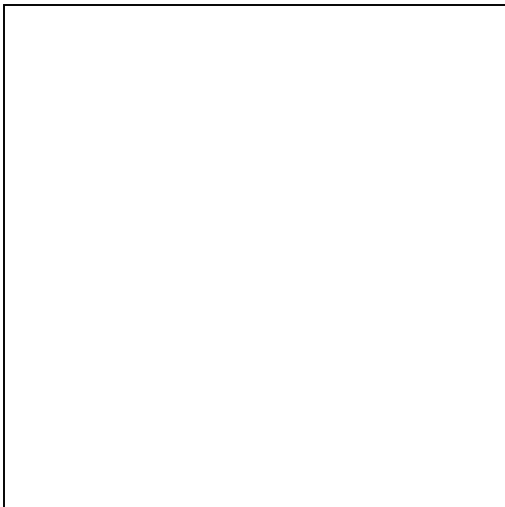
Find the **a** matrix for the left hand column of the structure. From the figure we get the value of the six element deformations:

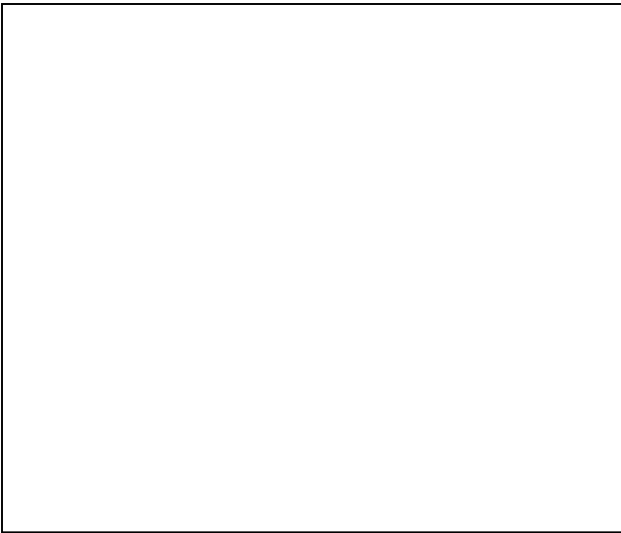
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

These values come from just taking the appropriate values from the displaced shapes. The values become column number one of the **a** matrix for this element. From the other displaced shapes we can fill out the other columns. The final matrix for the left column becomes:

$$\mathbf{a} \text{ (left column)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Using the same procedure we can get the **a** matrices for the other two elements.





The terms in the **a** matrix for the slanted column need to be explained. The second column is from the second displaced shape. Note that the pinned end is free to move when a displacement is applied. We can use slope deflection to find the rotation. Since we know the moment at a pin is equal to zero, we can write:

$$0 = M_{dc} = \frac{2EI}{L_{dc}} \left(2\theta_d + \theta_c + \frac{3\delta}{L_{dc}} \right)$$

solving for Θ_d in terms of Θ_c , knowing that $\delta=0$, we get:

$$\theta_d = \frac{\theta_c}{2}$$

Column number three comes from the third displaced shape. We know from similar triangles that the perpendicular displacement has the value 1.166. We also need to find the rotation at d. Again using the slope deflection equation and the fact that the moment equals zero, we can solve for the rotation Θ_d . Note that Θ_c is zero by definition since it is a DOF and must be held to zero when a unit displacement for δ is given. Again from the slope deflection formula, equation 5.12, we get;

$$2\theta_d = \frac{3\delta}{L_{dc}}$$

Solving for Θ_c gives;

$$\theta_d = \frac{3\delta}{2L_{dc}}$$

The definition of the **a** matrix is:

$a_{ij} =$ The displacement at element DOF i due to a unit displacement at global DOF j with all other global DOF equal to zero. (NOTE: there is one matrix for each element.)

