Consistent Deformations Derivation

Objectives:

1. Define consistent deformation variables
2. State consistent deformation procedure steps

Consistent deformations is based on compatibility. Compatibility states that a structure can deform in any shape provided the displacements conform to the boundary conditions and the material restrictions (i.e. the material can not tear apart).

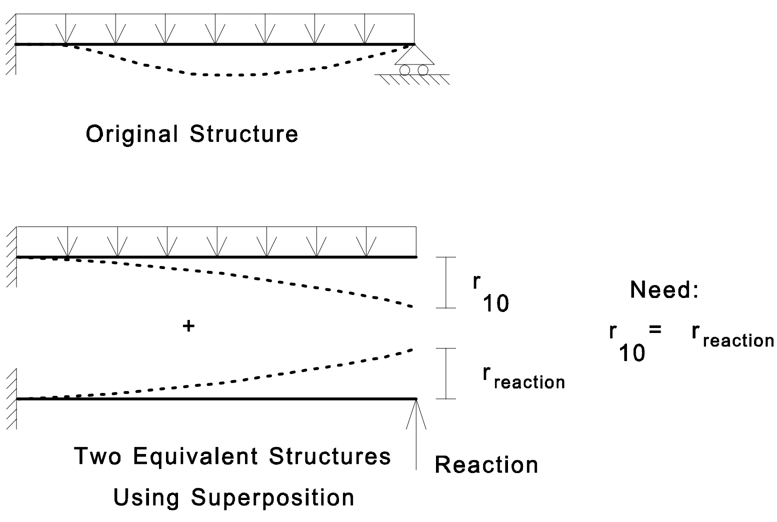
Consistent deformations process:

- Remove all the redundant reactions and forces (i.e. add hinges) until it becomes determinate.
- Then apply forces at the removed redundants to put the structure back into a compatible displacement.

Example, take the uniformly loaded propped cantilever beam.

One redundant (or extra) support.

1. Remove the prop or end support we have a cantilever beam.
2. Apply a force at the point where the support was removed.
3. Need to calculate the force needed to push the tip back up to the support.

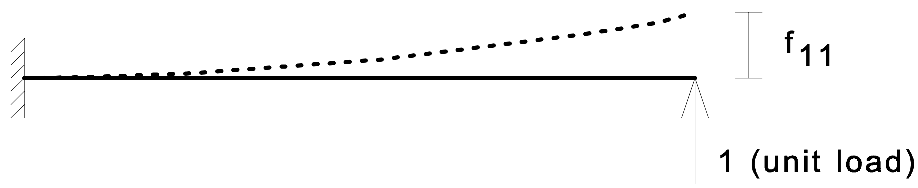


Derivation of Consistent Deformation Method

The tip deflection under uniform load is:

$$r_{10} = \frac{wL^4}{8EI}$$

The tip deflection due to a unit force at the end of the beam:



Displacement Caused by Unit Load

$$f_{11} = \frac{L^3}{3EI}$$

Apply a tip force equal to the ratio of the deflection without the prop to that of the unit force.

$$R_{prop} = \frac{\frac{wL^4}{8EI}}{\frac{L^3}{3EI}}$$

Definitions:

r_i is the final displacement at removed redundant i. This is usually zero but can also be a final support displacement such as a support settlement.

r_{is} is the displacement at redundant i on the determinate base structure due to settlement of the supports. This is only a rigid body displacement and only requires geometry to calculate.

r_{it} is the displacement at redundant i on the determinate base structure due to temperature.

r_{ie} is the displacement at redundant i on the determinate base structure due to initial imperfections.

r_{i0} is the displacement at redundant i on the determinate base structure due to the applied loading.

f_{ij} is the displacement at redundant i on the determinate base structure due to a unit force at redundant j.

X_j is the unknown redundant force at redundant j. This can be a support reaction or an internal force like a moment or an axial force.

The second index refers to the load applied that caused the deflection.

Consistent Deformation Solution Procedure

1) Remove all redundant forces to form a determinate base structure. This can involve removing supports, adding hinges or cutting members. The choice of which forces to remove is arbitrary, however clever choices can make the calculations simpler.

2) Calculate the displacements at each of the removed redundants due to all the applied loading. This should include loads, temperature and settlement. Each type of loading is considered to be its own load case. Therefore, applied loads are considered as load case 0. Temperature is considered as load case t, etc.

3) Apply a unit force at each redundant, one at a time, and find the displacements at all of the other removed redundants. Each redundant creates a separate load case, designated as load case j, where j corresponds to the redundant at which the load is applied. This will develop the terms f_{ij}. The applied unit load is at redundant j. The calculated displacements are at all other redundants i. Each load case will fill out a column of the **F** matrix. Applying the unit load at another redundant will develop another column of the **F** matrix.

4) Apply the compatibility equation:

$$r_i = r_{is} + r_{it} + r_{ie} + r_i0 + \sum_{j}^{\text{all redundants}} f_{ij} X_j$$

There is one compatibility equation for each removed redundant. This will create a set of simultaneous equations with X_j's as the unknowns. Solving the set of equations gives the values for the removed redundant forces.