Direct Stiffness Solution Procedure

Objectives:

1) List the Direct Stiffness Solution Procedure
2) Calculate concentrated loads from element loads
3) Recover element forces from global displacements

The process can now be reduced to the following steps:

1) Define the structural (global) DOF (**r**)
2) Find the **a** matrix for each element. This is done by applying a unit displacement to each DOF, holding all others still, and find the corresponding element displacements. This fills a column of the **a** matrix for each displacement applied. Repeat until all columns of all **a** matrices are filled.
3) Sum $\mathbf{a}^T \mathbf{K}_e \mathbf{a}$ for all elements in the structure.
4) Form the load vector **R**.
5) Solve the set of equations $\mathbf{K}\,\mathbf{r} = \mathbf{R}$.
6) Recover the element forces.

Beam Element Loads

Conversion of distributed loads by matrix operations. Loads are first converted into the concentrated equivalent fixed end forces.

$$R = a^T S$$

This equation transforms the element end forces into the global loads.

$$S^f = -(\text{fixed end forces})$$

The equation to convert all element loads to global loads is:

$$R = \sum_{\text{all elements}} a^T S^f$$

Beam Element Force Recovery

The final step of the procedure is force recovery. This step takes the final structural displacements, **r** and converts them into element end forces.

$$S = K_e\ v$$

If we substitute for **v** we get:

$$S = K_e a\ r$$

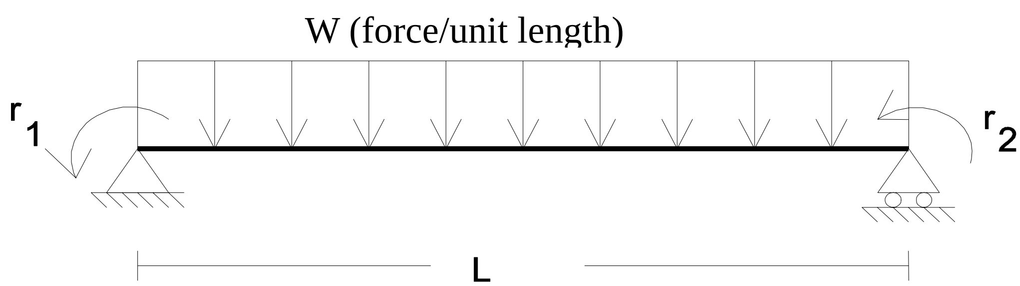
This equation directly converts the global DOF displacements into element forces in the local element directions. Re-writing:

$$K = \sum_{\text{all elements}} a^T FT$$

where

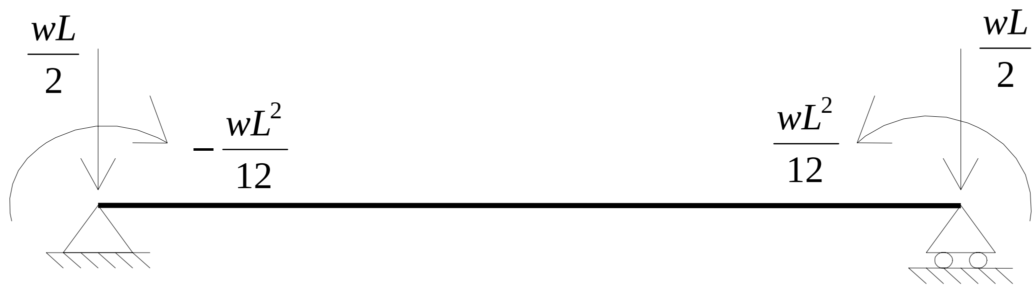
$$FT = K_e a$$

A correction is needed which is derived below:



Uniform Load Example

Following the standard procedure, we convert the distributed load to the equivalent loads:



Equivalent Concentrated Loads

Next, form the stiffness and the final equations:

$$\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Solving this set of simultaneous equations for the displacements we get:

$$r_1 = -r_2 = -\frac{wL^3}{24EI}$$

Performing the final step of force recovery:

$$S = \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} -r_1 \\ r_2 \end{bmatrix}$$

Substituting for $\mathbf{r_1}$ and $\mathbf{r_2}$ we get:

$$S = \begin{bmatrix} -\frac{wL^2}{12} \\ \frac{wL^2}{12} \end{bmatrix}$$

But we know that the moments on the end of a simply supported beam are equal to zero. To correct these recovered forces, we just need to subtract off the equivalent loads applied to the structure and add back the original distributed load. The final equation is:

$$S = K_e a\, r - S^f = FT\, r - S^f$$

Final Direct Stiffness Solution Procedure

The final solution procedure can be summarized as:

1) Define the Global DOF, **r**, for the structure.
2) Find the **a** matrix for each element in the structure.
3) Sum $\mathbf{a}^T \mathbf{K}_e \mathbf{a}$ for all elements.
4) Form the load, $\mathbf{R} = \mathbf{R}_{concentrated} + \Sigma\, \mathbf{a}^T \mathbf{S}^f$
5) Solve $\mathbf{K}\, \mathbf{r} = \mathbf{R}$
6) Recover the element force, $\mathbf{S} = \mathbf{K}_e\, \mathbf{a}\, \mathbf{r} - \mathbf{S}^f$