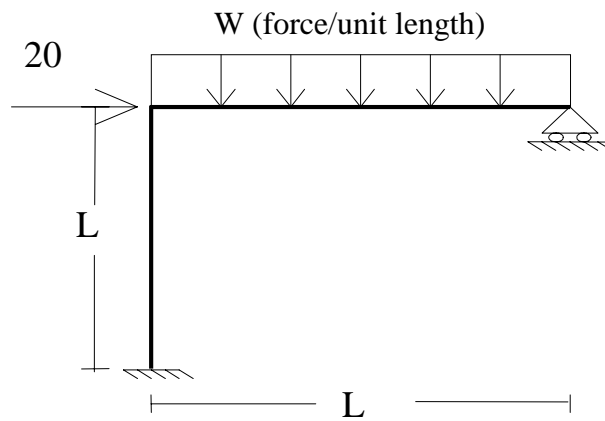


Direct Stiffness Frame Example

Objectives:

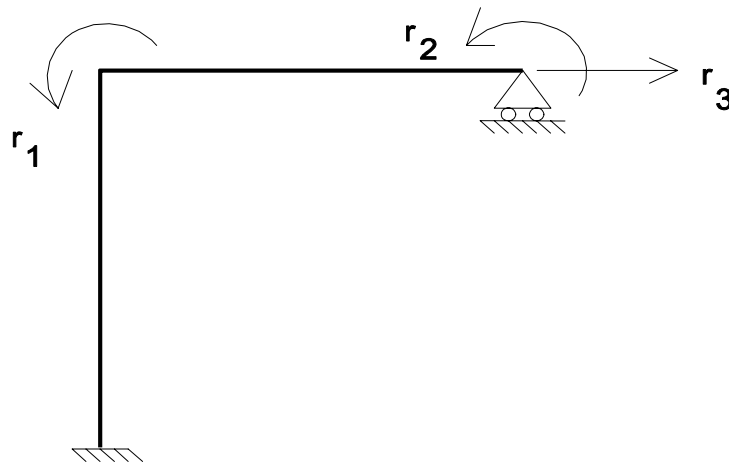
- 1) Apply the direct stiffness procedure to a structure
- 2) Form the required matrices
- 3) Solve for structural displacements and forces

Solve the following structure for displacements and forces using the direct stiffness method.



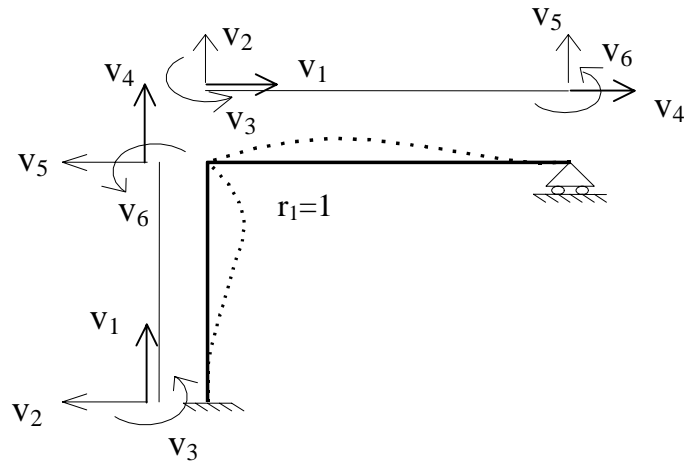
Direct Stiffness Example

The first step is to choose the global DOF.

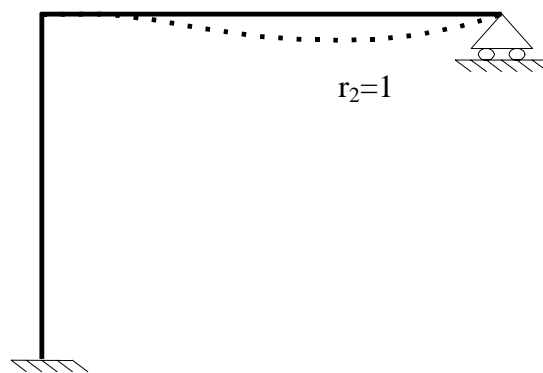


DOF for Direct Stiffness Example

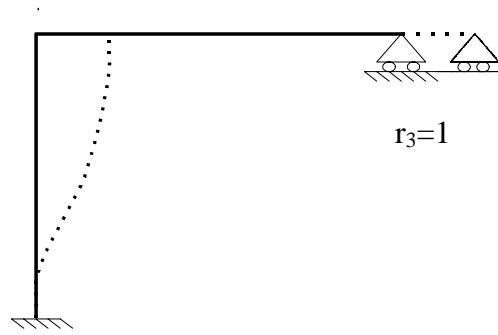
The second step is to form the **a** matrices for each element.



Unit Displacement at DOF 1



Unit Displacement at DOF 2



Unit Displacement at DOF 3

The element coordinate systems are chosen and shown on the first displaced shape.

$$a_1 = \begin{bmatrix} \\ \\ \end{bmatrix} \quad a_2 = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Multiply and sum the results for each element to get K.

$$K = a_1^T K_e a_1 + a_2^T K_e a_2 = K_1 + K_2$$

Where the subscript again represents which element produced the contribution to the global stiffness.

$$K_1 = \begin{bmatrix} \frac{4EI}{L} & 0 & \frac{6EI}{L^2} \\ 0 & 0 & 0 \\ \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} \end{bmatrix}$$

$$K_2 = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The final stiffness matrix is the sum of these two element contributions and is:

$$K = \begin{bmatrix} \frac{8EI}{L} & \frac{2EI}{L} & \frac{6EI}{L^2} \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} \end{bmatrix}$$

A diagonal term in a stiffness matrix says there is an element connected to that DOF. In order for a structure to be stable there should always be a non-zero diagonal in the global stiffness. The element stiffness may however have zero diagonal stiffness terms.

An off diagonal term in a stiffness matrix means that an element is linking the two DOF where the term is located. For example, the \mathbf{K}_{12} has a non-zero value. This means that some element (or elements) connects DOF 1 and DOF 2.

Next we need the load vector \mathbf{R} . First consider the concentrated loads acting directly at the DOF.

$$R_{concentrated} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

Next, include loads within the span of an element.

$$S_2^f = \begin{bmatrix} \\ \end{bmatrix}$$

Using the conversion equation, we get the global load vector contribution:

$$R_2 = a_2^T S_2^f$$

If we multiply out the matrices we get:

$$R_2 = \begin{bmatrix} \\ \end{bmatrix}$$

Note that the element contributes directly to the first two DOF. The final form of the load vector is:

$$R = R_{concentrated} + R_1 + R_2$$

$$R = \begin{bmatrix} -\frac{wL^2}{12} \\ \frac{wL^2}{12} \\ 20 \end{bmatrix}$$

Using MathCad – you can easily perform the required multiplications.