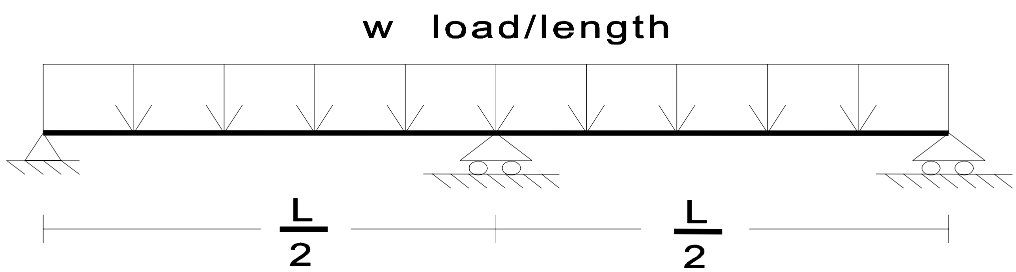
Consistent Deformations Beam Example

Objectives:

1. Set up the coordinate systems for virtual work in a beam
2. Calculate the moment equations for sections of a beam
3. Integrate using the Virtual work formulas to get displacements
4. Calculate the reactions for a 1-degree indeterminate beam structure

Example: One-degree indeterminate structure, two span continuous beam.

Step 1), is to create the determinate base structure by removing supports.



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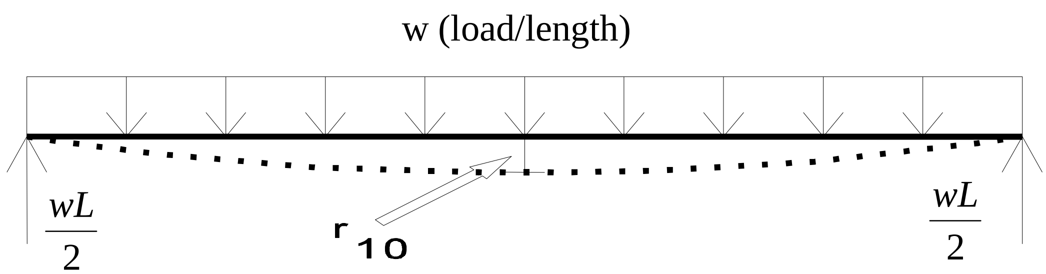
Possible determinate base structures:

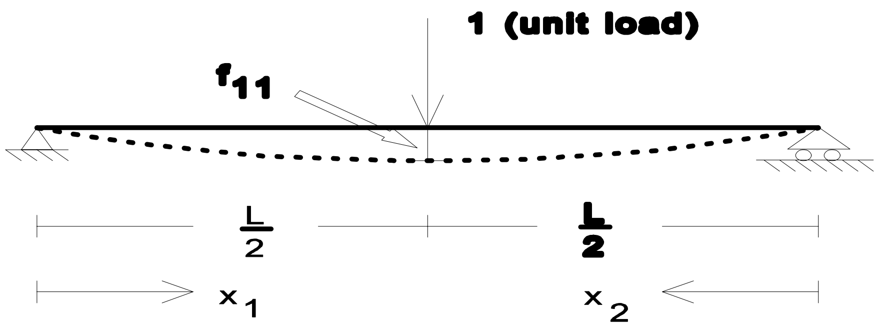
Choose below since displacement easier.



Load Case 0

Step 2) calculate the displacement at the center (where the redundant was removed) as a result of the applied loading.

The needed displacement is at the center of the structure, r_{10}. Using Virtual Work, the virtual load is:



Virtual Load and Load Case 1

The moment equations for the real loading are:

The moment equations for the virtual loading are:

$$\overline{M}_1 = \frac{x_1}{2}$$

$$\overline{M}_2 = \frac{x_2}{2}$$

The internal virtual work for r_{10} is given by:

The resulting displacement at the center after integration is:

$$r_{10} = \frac{5\,wL^4}{384EI}$$

Step 3) calculate the displacement at each removed redundant due to a unit load applied at all of the redundants, one at a time. (Shown as f_{11} above)

The applied unit load is also the virtual load required to calculate the displacements.

Using symmetry (hence 2x), the internal virtual work is:

The result of the integration is:

$$f_{11} = \frac{L^3}{48EI}$$

Step 4) enforce compatibility. In this case, $r_1 = r_{1t} = r_{1e} = r_{1s} = 0$, therefore we have:

$$0 = f_{11} X_1 + r_{10}$$

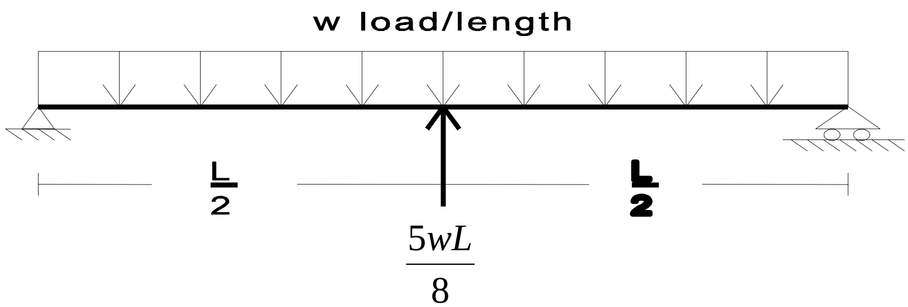
Substituting the values for the terms in the above equation we have:

Solving for X_1 we have:

$$X_1 = -\frac{5wL}{8}$$

Minus means the force X_1 is in the opposite direction as the unit load used for load case 1.

The final structure is:



Resulting Determinate Structure

You can use any method desired to determine displacements, rotations, moments, shears, etc.

The final support reactions are (subscripts refer to load case):

$$Reactions = Reactions_o + X_1 * Reactions_1$$

For this problem:

$$Reactions = \frac{wL}{2} + \left(\frac{5wL}{8}\right) * \frac{1}{2} = \frac{3wL}{16}$$