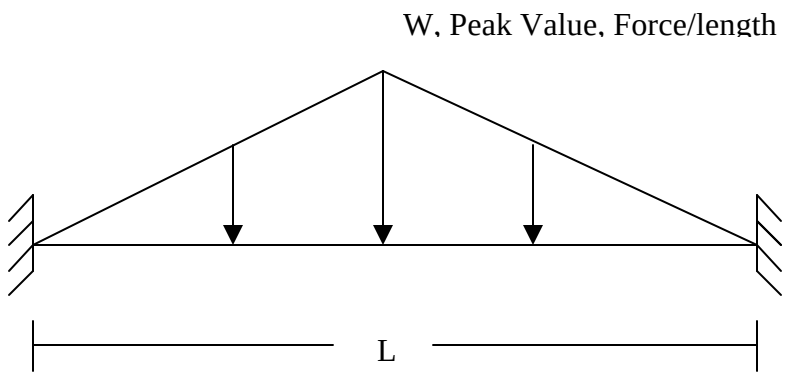
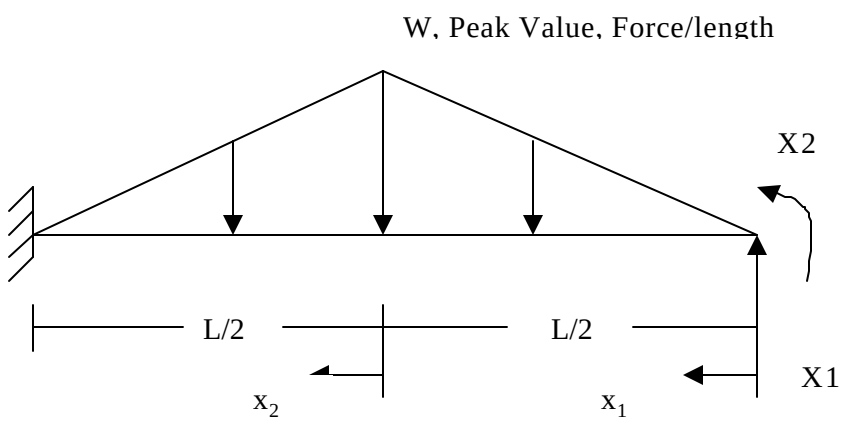
Homework #3, Triangular load Fixed End Force Solution

Given the following structure, find the reactions:



For the structure, create the determinate base structure:

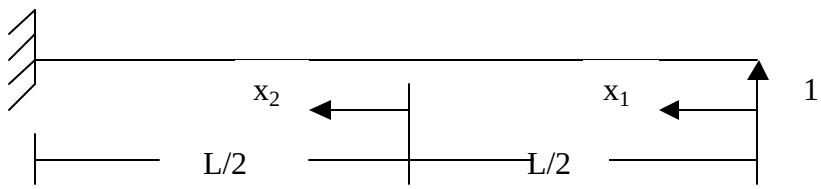


The Moment Equations are:

$$M_1 := \frac{w}{(L)} \cdot x_1 \cdot \left(\frac{x_1}{2}\right) \cdot \left(\frac{x_1}{3}\right)$$

$$M_2 := \left[\frac{w}{\left(\frac{L}{2}\right)} \cdot \left(\frac{L}{2} - x_2\right) \cdot x_2 \cdot \frac{x_2}{2} + \frac{w}{\left(\frac{L}{2}\right)} \cdot x_2 \cdot \left(\frac{x_2}{2}\right) \cdot \left(\frac{2 \cdot x_2}{3}\right) + w \cdot \frac{L}{2} \cdot \frac{1}{2} \cdot \left(\frac{L}{2} \cdot \frac{1}{3} + x_2\right) \right]$$

The Case 1 structure, unit load at redundant 1:

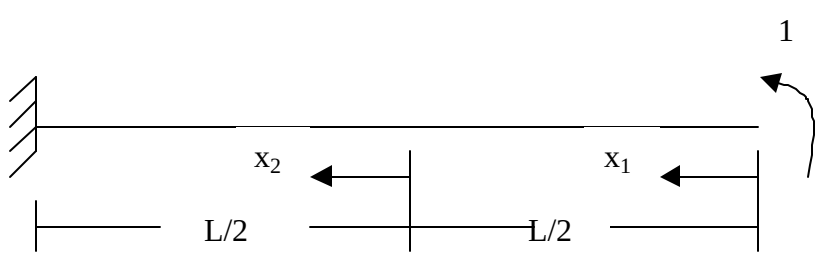


The Moment equations are:

$$M1_1 := -x_1$$

$$M1_2 := -\left(x_2 + \frac{L}{2}\right)$$

The Case 2 structure, unit moment at redundant 2:



The Moment equations are:

$$M2_1 := -1$$

$$M2_2 := -1$$

Integrating to get the required displacements:

$$\Delta_{10} := \int_0^{\frac{L}{2}} \frac{M_1 \cdot M1_1}{E \cdot I} dx_1 + \int_0^{\frac{L}{2}} \frac{(M_2 \cdot M1_2)}{E \cdot I} dx_2 \rightarrow \frac{-11}{192} \cdot L^4 \cdot \frac{w}{(E \cdot I)}$$

$$\Delta_{20} := \int_0^{\frac{L}{2}} \frac{M_1 \cdot M2_1}{E \cdot I} dx_1 + \int_0^{\frac{L}{2}} \frac{(M_2 \cdot M2_2)}{E \cdot I} dx_2 \rightarrow \frac{-7}{96} \cdot w \cdot \frac{L^3}{(E \cdot I)}$$

$$f_{11} := \int_0^{\frac{L}{2}} \frac{M1_1 \cdot M1_1}{E \cdot I} dx_1 + \int_0^{\frac{L}{2}} \frac{(M1_2 \cdot M1_2)}{E \cdot I} dx_2 \rightarrow \frac{1}{3} \cdot \frac{L^3}{(E \cdot I)}$$

$$f_{12} := \int_0^{\frac{L}{2}} \frac{M1_1 \cdot M2_1}{E \cdot I} dx_1 + \int_0^{\frac{L}{2}} \frac{(M1_2 \cdot M2_2)}{E \cdot I} dx_2 \rightarrow \frac{1}{2} \cdot \frac{L^2}{(E \cdot I)}$$

$$f_{22} := \int_0^{\frac{L}{2}} \frac{M2_1 \cdot M2_1}{E \cdot I} dx_1 + \int_0^{\frac{L}{2}} \frac{\left(M2_2 \cdot M2_2\right)}{E \cdot I} dx_2 \rightarrow \frac{L}{(E \cdot I)}$$

Set up final compatibiltiy equations:

$$F := \begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{pmatrix} \rightarrow \begin{bmatrix} \frac{1}{3} \cdot \frac{L^3}{(E \cdot I)} & \frac{1}{2} \cdot \frac{L^2}{(E \cdot I)} \\ \frac{1}{2} \cdot \frac{L^2}{(E \cdot I)} & \frac{L}{(E \cdot I)} \end{bmatrix}$$

$$F_{inv} := \begin{bmatrix} \frac{f_{22}}{\left(f_{11} \cdot f_{22} - f_{12}^2\right)} & \frac{-f_{12}}{\left(f_{11} \cdot f_{22} - f_{12}^2\right)} \\ \frac{-f_{12}}{\left(f_{11} \cdot f_{22} - f_{12}^2\right)} & \frac{f_{11}}{\left(f_{11} \cdot f_{22} - f_{12}^2\right)} \end{bmatrix} \rightarrow \begin{pmatrix} \frac{12}{L^3} \cdot E \cdot I & \frac{-6}{L^2} \cdot E \cdot I \\ \frac{-6}{L^2} \cdot E \cdot I & \frac{4}{L} \cdot E \cdot I \end{pmatrix}$$

$$\Delta := \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \end{pmatrix} \rightarrow \begin{bmatrix} \frac{-11}{192} \cdot L^4 \cdot \frac{w}{(E \cdot I)} \\ \frac{-7}{96} \cdot w \cdot \frac{L^3}{(E \cdot I)} \end{bmatrix}$$

$$X := F_{inv} \cdot \Delta \rightarrow \begin{pmatrix} \frac{-1}{4} \cdot L \cdot w \\ \frac{5}{96} \cdot w \cdot L^2 \end{pmatrix}$$